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CAPITAL ACCUMULATION, OVERLAPPING GENERATIONS AND DYNAMIC EFFICIENCY IN PENSION FUNDING

This paper considers dynamic efficiency and income distribution using an overlapping generation model of economic growth. The issues discussed within this framework intend to shed light on an ongoing debate in most developed countries: the reform in funding the retirement pension schemes. Private saving or compulsory distribution of income working independently of one another cannot achieve optimality. A flexible framework for policy action hinges on providing a mix of private saving and the adequate share of state sponsored security system.

Keywords: *Overlapping generations, consumption-efficiency, Golden Rule, "pay-as-you-go" security system*

1. Introduction

There is a lot of pressure in most developed countries to reform the funding of the pension scheme. The actual state sponsored redistribution pension system has come under strong criticism for the loss of its efficiency due to an aging and slowly growing population. In such a system, one would argue that there is an increasing burden put on the contribution of the young working population in funding the old and retired. The opponents to the actual *pay-as-you-go* system advocate the shift to private saving, and pension financing through trust funds, as to avoid the working population paying for the others.

This simplified reasoning involves a flaw. One can argue that in either system the pension payments will involve tapping into the national product of the same period. The working population will always provide the needs of the old generation by pro-

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ductive activity during the same period. The saving used in today's production activities is recovered (by savers) from tomorrow's output which is carried out by tomorrow's working population. Contrary to common belief, the saving payments made to pension funds are not stocked or "stowed" away from economic activity because national income is a flow variable realized in each period. An amount of capital invested today will grow only if there is a working population in the next period. No matter which system is at work, no generation does in fact fund its own retirement scheme.

This paper aims to address these issues in a simple and unified framework. A one-commodity "Diamond" [9] type life-cycle model is selected for this purpose. This choice is the one commonly used (but not exclusively) by the growing literature dealing with the subject. These models often consider a second asset, money, and feature multiple equilibria. From a quick survey, the main conclusions we draw are the following[1]: when capital accumulation is insufficient, increasing the saving invested in the financial market will boost economic growth. One can then argue that shifting to less *pay-as-you-go* funding, and therefore an increased share of pension financing through trust funds is a desirable outcome for growth. This proposition is valid when the constant rate of growth is driven by exogenous factors, such as technical progress, without any consideration of economic policy. Extending Romer's work [10], endogenous growth models dealing with the subject lead to ambiguous conclusions [1]. Concerning welfare outcomes, most models tend to prove that *pay-as-you-go* pension schemes are non-optimal, whereas empirical contributions suggest the opposite [5].

With the overlapping generation model as a framework, the paper argues that the debate opposing two financing pension systems is somewhat misleading. Neither private saving alone nor the *pay-as-you-go* pension funding can achieve optimality. By an adequate interpretation of the Golden Rule steady state, it is shown that the latter hinges on an intergeneration distribution mechanism. This outcome is feasible through a *"Rousseau"* type of *"social contract"* linking the old generation to the young one. When such solidarity cannot be accounted for, a distribution mechanism must then be implemented. In the Golden Rule steady state, welfare is optimal in both senses: the intertemporal spread of consumption is maximized over the life cycle of an individual, as well as the consumption of both generations over the same period. These conclusions lead to another result: an individual is indifferent as to the funding of the retirement pension system. One cannot choose one system or the other as the rate of return of the intergeneration distribution mechanism is equal to the rate of return of private saving. Therefore comparing pension systems in the Golden Rule steady state is misleading. We can, however, show that for all feasible steady states, the inequality between the two rates of return can go either way indicating a preference for one funding method over the other. This is a good indication for implementing economic policy. Private saving or compulsory distribution of income working

[1] See Artus P. [1993] for a survey of these conclusions, given in the reference by [1].

independently of one another cannot achieve optimality. Therefore funding the retirement scheme exclusively by the distribution mechanism is not optimal because capital accumulation is always driven by private saving.

A flexible framework for policy action hinges on providing a mix of private saving and the adequate share of intergeneration distribution of income. By introducing a taxation/distribution mechanism, the paper shows that when "over-accumulation" prevails and therefore the programme is *consumption-inefficient,* then the *pay-as-you-go* pension scheme helps to absorb excess saving. The policy should adjust the intergeneration transfer of income from the young towards the old. When capital accumulation is insufficient, the programme is *consumption-efficient*, but falls short of its Golden Rule value, then economic policy acts as to boost the insufficient saving. The conclusion now features a transfer of wealth in the opposite direction. Taxing the income of the old generation will improve capital accumulation and increase welfare.

The paper is organized as follows. Section 2 sets the life cycle model. Section 3 is a reminder of the Golden Rule and *consumption-efficiency.* In Section 4, the main results are discussed. The concluding section examines some of the implications in modelling behaviour in a dynamic setting.

2. The model

2.1. Intertemporal maximization behaviour

Let us consider a discrete-time life-cycle model when individuals live exactly two periods and maximize their intertemporal utility function. At any period of time, there are two types of consumers : the "young" and the "old". For simplicity , we assume that the utility function is of the Cobb–Douglas type :

$$U(c_1, c_2) = c_1^{\beta} c_2^{1-\beta}, \text{ and } 0 < \beta < 1 \tag{1}$$

where c_1 and c_2 stand for consumption during the first and second periods, respectively.

We also assume, on the supply side, a single good competitive economy. Output is produced by a constant return to scale technology, satisfying all the *Inada* regularity conditions.

The intensive form of the production function, relating the per capita output y to the capital–labor ratio k, is given by: $y = f(k)$ with a positive, but decreasing marginal product of capital $f'(k) > 0$ and $f''(k) < 0$.

Without loss of generality, we consider a Cobb–Douglas production function such that:

$$F(K;L) = K^{\alpha} L^{1-\alpha} \text{, such as } 0 < \alpha < 1, \tag{2}$$

and

$$f(k) = F\left(\frac{K}{L}, 1\right) = k^{\alpha} .$$

At any period, there are two overlapping generations, the young active workers and the old retired generation, as illustrated by Fig. 1.

period	1	2	2	4	5
*generation*1	c_1	c_2	$-$	$-$	$-$
*generation*2		c_1	c_2	$-$	$-$
*generation*3			c_1	c_2	$-$

Fig. 1

During the first period of his/her life, while being young, a worker's only source of income is his/her wage which depends on the capital–labor ratio of the first time period:

$$w_1 = f(k_1) - k_1 f'(k_1) . \tag{3}$$

This worker consumes c_1 and saves s_1; so, $s_1 = w_1 - c_1$. This saving is invested and earns a yield equal to the marginal product of capital of the next period. Bequest considerations are ignored such that the individual's consumption, while being old and retired, equals the income of this second period: $c_2 = s_1 f'(k_2)$.

For simplicity we consider a circulating-capital model. With the rate of depreciation of capital equal to 1, the net marginal product (of capital) is equal to the commodity interest rate:

$$f'(k_2) - 1 = r_2 \text{, or also } f'(k_2) = (1 + r_2) . \tag{4}$$

Therefore, the second period consumption is: $c_2 = s_1(1 + r_2)$.

Solving the life-cycle utility maximization problem (Problem I)

$$\max : U = c_1^{\beta} c_2^{1-\beta}$$

subject to the income constraints:

$$w_1 = f(k_1) - k_1 f'(k_1),$$
$$c_1 = w_1 - s_1,$$
$$c_2 = s_1(1 + r_2)$$

yields the solution of the optimal saving rate:

$$s_1 = (1-\beta)\, w_1 \,. \tag{5}$$

This is a linear function which depends exclusively on the first period income: the exogenously given wage rate. It is therefore independent of the expected income of the second period.

2.2. Capital accumulation and the steady state

We now turn to the capital accumulation dynamics when individuals have a saving behaviour, for all future generations, as the one described above. We postulate a constant growth rate "*n*" of the population such that:

$$L_{t+1} = (1+n)L_t \,. \tag{6}$$

Accordingly, total saving, for any time period, is given by:

$$\sum_{i=1}^{L_1} s_{i1} = L_1 s_1 = L_1(1-\beta)w_1 = S_1 \,.$$

This saving is converted through investment to form the capital used during the next period. Therefore, the capital–labor ratio of any period 2 of the life cycle is: $k_2 = \dfrac{S_1}{L_2}$.

By substituting the values of S_1 and L_2, we get:

$$k_2 = \frac{1-\beta}{1+n} w_1 \,. \tag{7}$$

Reverting to the per capita production function (equation (2)), we can write the wage rate (equation (3)) as:

$$w_1 = (1-\alpha)k_1^{\alpha} \,. \tag{8}$$

Equations (7) and (8) combine, for any two successive periods of time t and $t + 1$ to give:

$$k_{t+1} = \frac{(1-\beta)(1-\alpha)}{1+n} k_t^{\alpha} \,.$$

This is a simple nonlinear difference equation in k_t for this perfectly competitive model with perfect foresight and intertemporal utility maximization by economic agents. It may readily be shown that the capital–labor ratio k, starting from any initial condition k_0, converges to its steady state value k^* as $t \to \infty$:

$$\lim_{t\to\infty} k_t = k^* = \left[\frac{(1-\beta)(1-\alpha)}{1+n}\right]^{\frac{1}{1-\alpha}}. \tag{9}$$

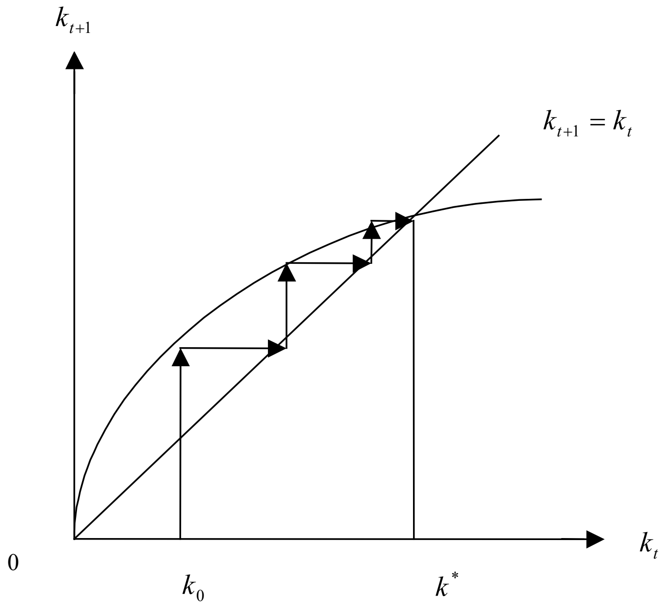

Fig. 2

Because the model is stable, the interest rate also converges to its steady state value. Using equation (4): $r_t = f'(k_t) - 1$, enables us to set:

$$\lim_{t\to\infty} r_t = r^* = f'(k^*) - 1 = \frac{(1+n)\alpha}{(1-\alpha)(1-\beta)} - 1. \tag{10}$$

Does this capital accumulation process yield a steady state which is *consumption-inefficient*, in the sense that everyone can be made better off by having higher consumption in every time period? Equation (10) has a key role in providing an answer to such a question. Before doing so, let us turn briefly to discussing the problem underlying the *consumption-inefficient* outcome.

3. The Golden Rule and the consumption efficiency

3.1. Capital accumulation and *consumption-efficiency*

The concept of economic efficiency we are looking at here involves comparison of alternative consumption paths over an infinite time horizon.

For models with one primary factor–labor- (and excluding joint production) and constant returns to scale, the steady-state equilibrium value of every variable may be regarded as a function of the steady-state commodity interest rate. This result follows from P. Samuelson's *Non-substitution Theorem.* Thus in a world of identical well-behaved preferences, it is quite a general proposition to write per capita utility as a function of the interest rate: $u = u(r)$.

In a one-commodity model, consumption is the difference between output and investment. Per capita consumption is given by its steady state expression as: $c = f(k) - (1+n)k$, where k is the steady state value of the capital–labor ratio (we also assume a depreciation rate equal to 1).

So, $k = k(r)$ and $u = u[c[k(r)]]$.

From among all stationary states, how can we select those capital accumulation programs that increase utility, or better, provide the highest level of welfare?

The behaviour of $u(r)$ is seen by calculating:

$$\frac{du(r)}{dr} = \frac{du}{dc}\frac{dc}{dk}\frac{dk}{dr} = \frac{u'}{f''}[f'(k) - (1+n)] .$$

Combining the regularity conditions; $u' > 0, f'' < 0$ with equation (4): $f'(k) - 1 = r$, leads to the following result:

$$\operatorname{sgn}\frac{du}{dr} = -\operatorname{sgn}(r-n) \text{, across steady state equilibria.}$$

– Utility deriving from consumption is maximised if $r = n$. This is the Golden Rule of capital accumulation.

– What can we say when $r > n$? In this one commodity model, saving and capital accumulation are identical, therefore lower values of the interest rate are unambiguously associated with higher values of the steady-state capital–labor ratio k (recall that $f'(k)$ is a decreasing function of k). Thus for $r > n$, smaller values of r also imply higher levels of utility, as shown in Fig. 3. Therefore, this program is said to be *consumption-efficient.*

– For $r < n$, there is already too much saving. Therefore additional saving which implies still lower values of r and larger values of k, results in decreasing the steady-state level of utility, as Fig. 3 shows. There is over-accumulation due to excess saving, so the steady state is *consumption-inefficient.*

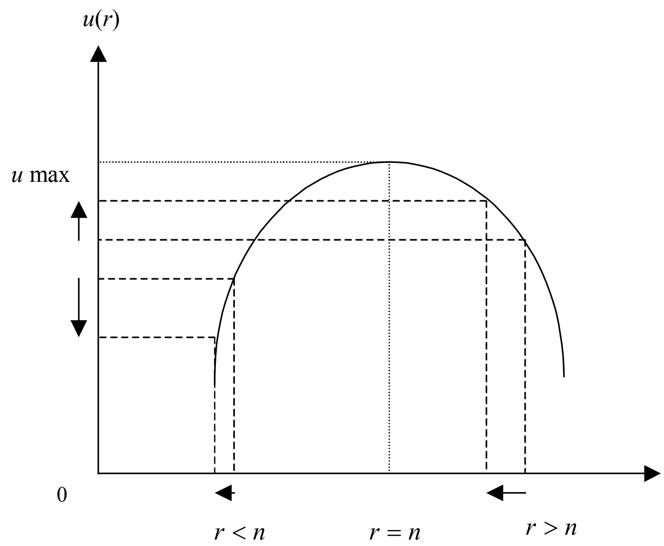

Fig. 3

The Golden Rule of capital accumulation serves as a benchmark to distinguish between the steady states equilibria having "too much" capital and those that do not.

A programme is *consumption-inefficient* if $r < n$.

3.2. *Consumption-inefficiency* in the life-cycle model

Let us revert to equation (10) of our life-cycle model of Section 2. Dynamic inefficiency occurs whenever:

$$r^* = \frac{(1+n)\alpha}{(1-\alpha)(1-\beta)} - 1 < n \,.$$

Therefore, when: $\dfrac{\alpha}{(1-\alpha)(1-\beta)} < 1$, or equivalently when $\dfrac{\alpha}{(1-\alpha)} < 1-\beta$.

A simple numerical example suggests that if $\alpha = \frac{1}{4}$ then the condition holds for $\beta < \frac{2}{3}$.

From the utility function in equation (1) as well as from equation (5), β is the fraction of the wage income devoted to consumption during the first period of an individual's life cycle. *Consumption-inefficiency* is due to excess saving. This is an important conclusion in the sense that we must recognize the theoretical possibility that dynamic inefficiency occurs despite perfect competition and intertemporal utility maximization in a perfect foresight world.

In a more practical sense, the result points out the possibility of *consumption-inefficiency* when agents rely on private saving to pay for consumption during their old age period of life.

Is there an alternative way to transfer wealth that provides a *consumption-efficient* steady state?

4. The intergeneration distribution solution

4.1. The Golden Rule and welfare criterion

The most frequently used criterion to evaluate welfare in an overlapping generation model is given by P. Samuelson [12]. It suggests choosing from among feasible stationary states the one that maximizes intertemporal utility. What is in fact suggested is an intergeneration distribution mechanism.

To see this, let us write the maximization problem: $\max U(c_1, c_2)$ subject to the constraint of the equation of balance between output, on the one hand, and consumption and investment, on the other, as:

$$K_t^{\alpha} L_t^{1-\alpha} = L_t c_{1t} + L_{t-1} c_{2t} + K_{t+1},$$

or in per capita form:

$$k_t^{\alpha} - (1+n)k_{t+1} = c_{1t} + \frac{1}{1+n} c_{2t}.$$

This is the balance between the net product and the consumption of the "young" and "old" generations at any period in time.

In steady state $k_t = k_{t+1}$. Aggregate per capita consumption is given by the steady-state net output: $c = k^{\alpha} - (1+n)k$.

Let us consider the transfer of income between generations. We assume that every young worker is given the net output: $k^{\alpha} - (1+n)k$.

Let us also assume that this representative agent is willing to transfer to the retired generation an amount equal to γ such that any old individual acquires: $(1+n)\gamma$.

While being young any individual's consumption possibilities are given by:

$$c_1 + \gamma = k^{\alpha} - (1 + n)k.$$

Once retired, he/she will benefit from transfers made by the younger generation. His/her consumption is then equal to: $c_2 = (1+n)\gamma$. Therefore, at any period in time along a steady state, per capita consumption in the economy is still given by the net per capita output:

$$c = c_1 + \frac{1}{1+n} c_2 = k^{\alpha} - (1+n)k\ .$$

Therefore, Samuelson's criterion scales down to choose the capital–labor ratio k that maximizes net product and then select the best intergeneration distribution pattern.

Maximizing c yields the Golden Rule capital–labor ratio as a solution:

$$\frac{dc}{dk} = 0 \Rightarrow k^{**} = \left(\frac{\alpha}{1+n}\right)^{\frac{1}{1-\alpha}}. \tag{11}$$

In addition, the maximized net output is given by $(1-\alpha)k^{\alpha}$, when k is set equal to k^{**}.

Selecting the distribution pattern is equivalent to maximizing welfare subject to the optimally chosen net output, that is:

$$\max U = c_1^{\beta} c_2^{1-\beta}$$

subject to: $c_1 + \dfrac{c_2}{1+n} = (1-\alpha)k^{\alpha}$.

Solving yields the arbitrage condition:

$$\frac{c_2}{c_1} = \frac{(1-\beta)}{\beta}(1+n)\ . \tag{12}$$

Equation (12) is an individual intertemporal maximization condition as well as an optimal intergeneration distribution rule. We can readily provide a solution for consumption and for the distribution parameter γ as:

$$c_1 = \beta(1-\alpha)k^{\alpha}\ ,\quad c_2 = (1-\beta)(1-\alpha)(1+n)k^{\alpha} \ \text{ and } \ \gamma = (1-\beta)(1-\alpha)k^{\alpha}\ . \tag{13}$$

This outcome is feasible through a "Rousseau" type of *"social contract"* linking the "young" and "old" generations. When such solidarity between generations does prevail, then some sort of regulation must be implemented by public intervention in the distribution mechanism. This can be obtained through a compulsory *pay-as-you-go* pension policy.

Comparing this solution with the one obtained with the private saving capital–labor ratio of equation (9), we can readily notice that:

$$k^* = k^{**}, \text{ if } (1-\beta)(1-\alpha) = \alpha, \text{ or if } \frac{\alpha}{1-\alpha} = (1-\beta),$$

which is equivalent to $r = n$, precisely the Golden Rule condition for capital accumulation.

The distribution mechanism based on the intergeneration transfer of income at any time is optimal only if the capital–labor ratio maximizes net per capita output. The latter depends on the saving rate and we know from the earlier discussion that private saving behaviour does not necessarily lead to the Golden Rule capital–labor ratio. Funding the retirement scheme exclusively by the distribution mechanism is not optimal because capital accumulation is determined by private saving. Therefore, any system, private saving or compulsory distribution of income, working independently of one another cannot achieve the Golden Rule steady state.

4.2. Alternative pension systems and the individual choice

Let us consider an individual who is free to choose between alternative pension systems. He/she can either rely exclusively on private saving or pay a contribution τ taken from his/her first period wage, and receive a pension θ_2 when retired.

If we assume, for simplicity, that his/her first period consumption $c_1 = \bar{c}$ is exogenous, the economic agent will only care about the expected consumption of the second period of his/her life cycle while choosing between the two systems, that is:

$$\max E[U(c_1, c_2)], \text{ with } c_1 = \bar{c} \text{ exogenous, is equivalent to } \max E[V(c_2)].$$

Even without a second asset in the economy, such as money, the consumer problem scales down to a portfolio choice. He/she will compare the rate of return of private saving to that of the distribution pension in order to make a decision (Problem II):

$$\max E[V(c_2)],$$

subject to:

$$c_1 = (1-\tau)w_1 - s_1,$$
$$c_2 = (1+r_2)s_1 + \theta_2^e,$$

where θ_2^e is the expected complementary income.

We can readily show that θ_2^e depends on population growth and on the wage rate: $\theta_2^e L_1 = \tau w_2 L_2$, and the distribution parameter is expressed as: $\theta_2^e = \tau w_2 \frac{L_2}{L_1} = \tau w_2 (1+n)$.

In a general setting, we may consider that the future wage increases because of technical progress, given by the parameter μ, such that: $\frac{w_2}{w_1} = (1+\mu)$.

In a perfect foresight framework, excluding uncertainty of population growth and technical progress, the payment accruing to an individual who subscribes to the social security system is:

$$\theta_2 = \tau(1+n)(1+\mu)w_1 .$$

The rate of return ρ of the contribution to the social security system is given by:

$$\rho = \frac{\tau(1+n)w_2 - \tau w_1}{\tau w_1} = (1+n)(1+\mu) - 1, \text{ therefore } \rho \approx n + \mu .$$

Any agent will compare the rate of return ρ to the commodity interest rate r before making a decision: $r \le n + \mu$ or $r \ge n + \mu$.

In our initial problem, we assumed that $\mu = 0$.

– If $r > n$, which is the case of a *consumption-efficient* programme, private saving earns a higher rate of return. No agent is willing to pay contributions to the social security system: $\tau = 0$.

– If $r < n$, the *pay-as-you-go* system is more attractive. The solution is to set $\tau = 1$ and therefore the agent may drop private saving.

– The case of $r = n$ is an indeterminate solution.

The cases discussed above are *corner* solutions and much caution is recommended when they are used to make decisions. It is not clear, for instance, in the case where $r > n$ whether or not the saving policy will drive the economy to the Golden Rule steady state. These results are nevertheless good indications for the implementation of economic policy.

4.3. The economic policy

Using the same parameters τ and θ as policy instruments, we define:

– τ = the tax rate affecting the wages of the "young" generation, such that the tax revenue equals: $\tau w_t L_t$.

– θ_t = the fixed amount distributed to every member of the "old" generation, such that $\theta_t L_{t-1} = \tau w_t L_t$. Using equation (6), we can write θ as $\theta_t = (1+n)\,\tau w_t$.

Recall that this fixed amount is not the only income during the second period of the life cycle of any agent. Workers, we mean the "young" generation, continue to save and therefore earn an income in the next period. The intertemporal maximization problem changes as (Problem III):

$$\max U = c_1^{\beta} c_2^{1-\beta}$$

subject to:

$$w_1 = \text{marginal product of labour},$$

$$c_1 = (1-\tau)w_1 - s_1\,,$$

$$c_2 = (1+r_2)s_1 + \theta_2\,.$$

Solving it as in Section 2, gives the optimal saving rate:

$$s_1 = (1-\beta)(1-\tau)\,w_1 - \frac{\beta\theta_2}{1+r_2}\,. \tag{14}$$

When compared with equation (5), we notice that s_1 now depends on the policy parameters, and on the expected income of the next period: $\frac{\beta\theta_2}{1+r_2}$.

Providing solution for the steady state capital–labor ratio yields:

$$k^{***} = \left[\frac{(1-\beta)(1-\tau)(1-\alpha)}{(1+n)\left(1+\beta\tau\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{1-\alpha}}.$$

We can deduce the optimal taxation policy by setting k^{***} equal to its Golden Rule value: $k^{**} = \frac{\alpha}{1-\alpha}$, therefore, the optimal taxation rate is set to

$$\tau = (1-\beta) - \frac{\alpha}{1-\alpha}\,. \tag{15}$$

– If the equilibrium is the Golden Rule steady state, the capital–labor ratio equals k^{**}, therefore the taxation rate $\tau = 0$. Private saving behaviour yields a *consumption-efficient* steady state. There is no need for policy regulation. We have shown earlier that this may be a rather exceptional outcome.

– If $r < n$, and over-accumulation prevails, then the *pay-as-you-go* pension scheme helps to absorb excess saving through a positive taxation rate: $\tau > 0$. Hence, there is a transfer from the younger generation towards the older one over the same period in order to achieve optimality.

– Lastly, if $r > n$, the accumulation program is *consumption-efficient*, but still saving (by the young) is insufficient and could be increased in order to boost the future stream of consumption. We now must implement a negative rate of taxation $\tau < 0$; that is, the transfer should be operating from the old generation towards the younger one.

The overlapping generation model discussed above highlights the distribution issues underlying capital accumulation. On the one hand, *consumption-efficiency* occurs when agents rely on private saving to transfer wealth between the successive periods of their life cycle. This "*intra-generation"* distribution mechanism fails because it does not provide an adequate signal in guiding efficient allocation of resources for future generations. On the other hand, an intergeneration transfer of income is consistent with an optimal solution for capital accumulation only if private decisions involve, besides individual rationality, some amount of *"solidarity"* between generations. When such a *"social contract"* is not effective, then some sort of regulation must be implemented by public intervention. Taxation and distribution combine to fulfil the optimal equilibrium conditions. The policy hinges on providing an optimal mix of private saving and an appropriate share of a *pay-as-you-go* pension scheme.

5. Conclusion

The results and conclusions we are able to draw from the overlapping generation model depend crucially on the exogenous constant saving rate assumption. The economic agents transfer income over a short planning horizon while society evolves indefinitely trough time.

There are alternative ways to transfer wealth as suggested by optimization growth models. Following R.J. Barro [4], the representative agent, in intertemporal optimization models, is assumed to be a family or a group of individuals linked to each other through bequests. The time horizon is then infinite as individuals care about their utility and about their children's welfare. The utility function takes into account the discount rate δ, which represents the time preference, such that:

$$W = \int_0^\infty e^{-\delta t}\left[\frac{c^{1-\sigma}-1}{1-\sigma}\right]dt,$$

σ is the constant intertemporal elasticity of substitution.

Little work has been devoted to understanding the determinants of the discount rate and the elasticity of substitution. If we knew why some societies or countries are more or less impatient, or more willing to inter-temporally substitute consumption than others are, we would then know what determines long run growth. The parameters are always taken as given and, therefore, are not subject to policy action[2].

In optimal growth models, *consumption-inefficiency* due to excess saving does not occur. Nevertheless the Modified Golden Rule ($r = n + \delta$) solution yields a lower per capita consumption level than its original Golden Rule counterpart. In spite of their robust construction, which allows them to embrace a wide range of intricate problems, the optimal growth models are not automatically best suited to deal with the funding of pension schemes. The latter involve a finite time planning horizon for individuals and a necessarily infinite horizon at the level of society.

The overlapping generation model captures this fundamental feature of the problem. It also reveals the necessity of public intervention in order to restore consistency between rational decisions made by finite lived agents and long-run dynamics. The saving rate is allowed to vary by appropriately manipulating the policy instruments. The corrective fiscal policy is implemented in a flexible design featuring the transfer of income in either direction between generations. The pension system then hinges on an adequate balance between private saving and state-sponsored redistribution.

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Akumulacja kapitału, nakładające się pokolenia oraz dynamiczna efektywność w finansowaniu funduszu emerytalnego

W krajach rozwiniętych kładzie się duży nacisk na schemat reformy systemu emerytalnego. Dowodzi się konieczności dokonania zmian w sposobie oszczędzania indywidualnego w porównaniu do rozpowszechnionego sposobu *zliczania kosztów postępowania* (*pay-as-you-go*) w systemie ubezpieczeń społecznych. Problem ten dyskutowany jest za pomocą modelu o *nakładających się pokoleniach* (*overlapping generations model*). Z pracy wynika, że debata nad przeciwstawnymi sobie dwoma systemami finansowania emerytur jest w pewnym stopniu zwodnicza. Z jednej strony, *nieefektywność konsumpcji* (*consumption-inefficiency*) występuje w przypadku, gdy administratorzy polegają jedynie na transferze oszczędności prywatnych w czasie trwania cyklu życia. Ten „wewnątrzpokoleniowy" mechanizm podziału załamuje się z braku adekwatnych sygnałów efektywnej alokacji zasobów dla przyszłych pokoleń. Z drugiej strony, międzypokoleniowy podział dóbr jest zgodny z optymalnym rozwiązaniem jedynie wówczas – przy dodatkowym założeniu racjonalności indywidualnej – kiedy istnieje pewna doza „solidarności" między pokoleniami.

W przypadku braku upowszechnienia takiego „kontraktu socjalnego" polityka podziału musi być uprawomocniona przez „pośrednictwo państwa". W pracy wykazano, że dla wszystkich rozwiązań o stabilnej równowadze nierówność pomiędzy stopą zysku oszczędności prywatnych a stopą zysku, uzyskaną z mechanizmu podziału międzypokoleniowego, zależy od preferencji w wyborze sposobu gromadzenia środków. Postulowana jest elastyczność akcji politycznych, prowadzących do powiązania oszczędności prywatnych z odpowiednim udziałem państwa w sponsorowaniu schematu ubezpieczeń. W pracy pokazano, że poprzez wprowadzenie mechanizmu podatki/podział pomocnym w absorbowaniu przeważnie występującej nadwyżki akumulacji jest system *zliczania kosztów postępowania* (*pay-as-you-go*). Polityka sprzyja przy tym transferowi dochodów młodszych do starszych. Polityka ekonomiczna sprzyja także podniesieniu niedostatecznego oszczędzania w przypadku, kiedy akumulacja kapitału spada poniżej *złotej wartości* (*Golden Rule*). Z konkluzji wynika wybór transferu dobrobytu w przeciwnym kierunku. Opodatkowanie dochodów starzejących się pokoleń wpływa na polepszenie opieki społecznej.

Słowa kluczowe: *nakładające się pokolenia, efektywność konsumpcji, „złota wartość", system ubezpieczeń społecznych oparty na zliczaniu kosztów postępowania*